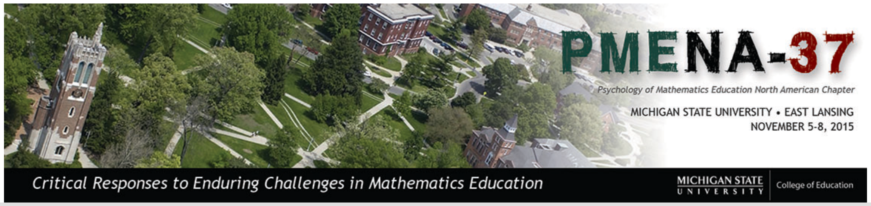
A Learning Progressions Approach to Early Algebra Research and Practice

Nicole Fonger
Univ. of Wisconsin-Madison

Ana Stephens
Univ. of Wisconsin-Madison

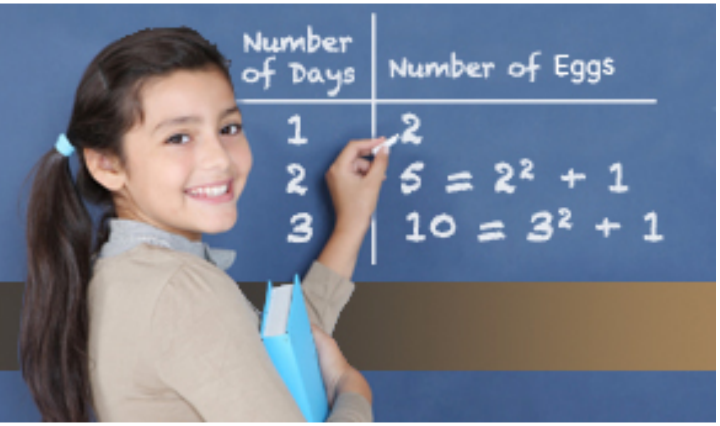
Maria Blanton
TERC

Eric Knuth
Univ. of Wisconsin-Madison



Project LEAP:

Learning through an Early Algebra Progression



TERC

Wisconsin Center for Education Research
SCHOOL OF EDUCATION
UNIVERSITY OF WISCONSIN–MADISON

The City College of New York

Maria Blanton
Lindsay Demers
Michael Eiland
Nicole Fonger
Emily Fyfe
Angela Gardner
José Gutiérrez
Hannah Kang
Eric Knuth
Despina Stylianou
Susie Strachota
Ana Stephens

NSF

ies INSTITUTE OF EDUCATION SCIENCES

The research is supported in part by the National Science Foundation under grants DRL-1207945, DRL-1219606, and DRL-1219605; and the U.S. Dept. of Education-IES Research Training Programs in the Education Sciences under grant no. R305B130007.

How might a child reason?

8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 – 24 = 47 – 24.

   a) Do you agree with Ian? Why or why not?

8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 - 24 = 47 - 24.

a) Do you agree with Ian? Why or why not?

No because 37+10=47 then he 47-24 which equals 23 not 47.

Operational

8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 - 24 = 47 - 24.

a) Do you agree with Ian? Why or why not?

Yes because 37+10-24 47-24 23 47-24=23

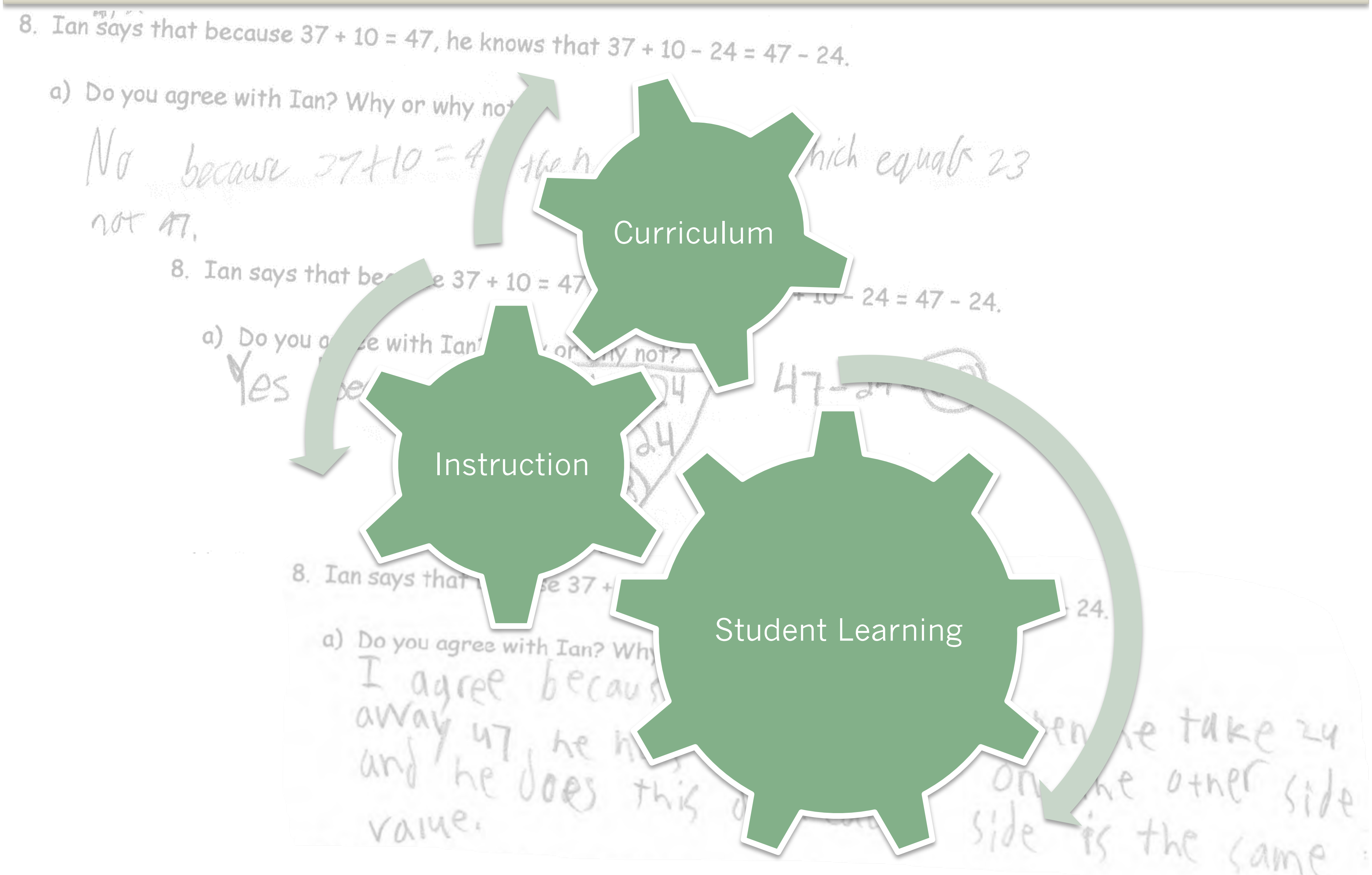
Relational-Computational

8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 - 24 = 47 - 24.

a) Do you agree with Ian? Why or why not?

I agree because 37+10=47 but when he take 24 away 47, he has to take 24 on the other side and he does this and each side is the same value.

Relational-Structural


8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 - 24 = 47 - 24.
Curriculum
Instruction
Student Learning

Research Goals

How can we support an integrated understanding of concepts of early algebra over time?

How can we characterize students' algebraic thinking and understanding of core concepts over time?

What is a coherent theoretical frame to support addressing these goals?

LEAP 1

- Development of Curricular Framework & Progression, Instructional Sequence (LEAP Intervention)
- Efficacy study, grade 3 intervention (Blanton et al., 2015)

LEAP 2

- Grade 3-5 quasi-experimental study of LEAP intervention
- Comparisons of growth in students' understanding per Levels of Sophistication over time

LEAP 3

- Teacher-led LEAP intervention, large, randomized longitudinal study
- Instructional supports for understanding and responding to students' understandings

Blanton, Stephens, Knuth, Gardiner, Isler & Kim (2015)

An Early Algebra Learning Progression

Curricular Framework & Progression

Assessments

Supports

Learning Progression

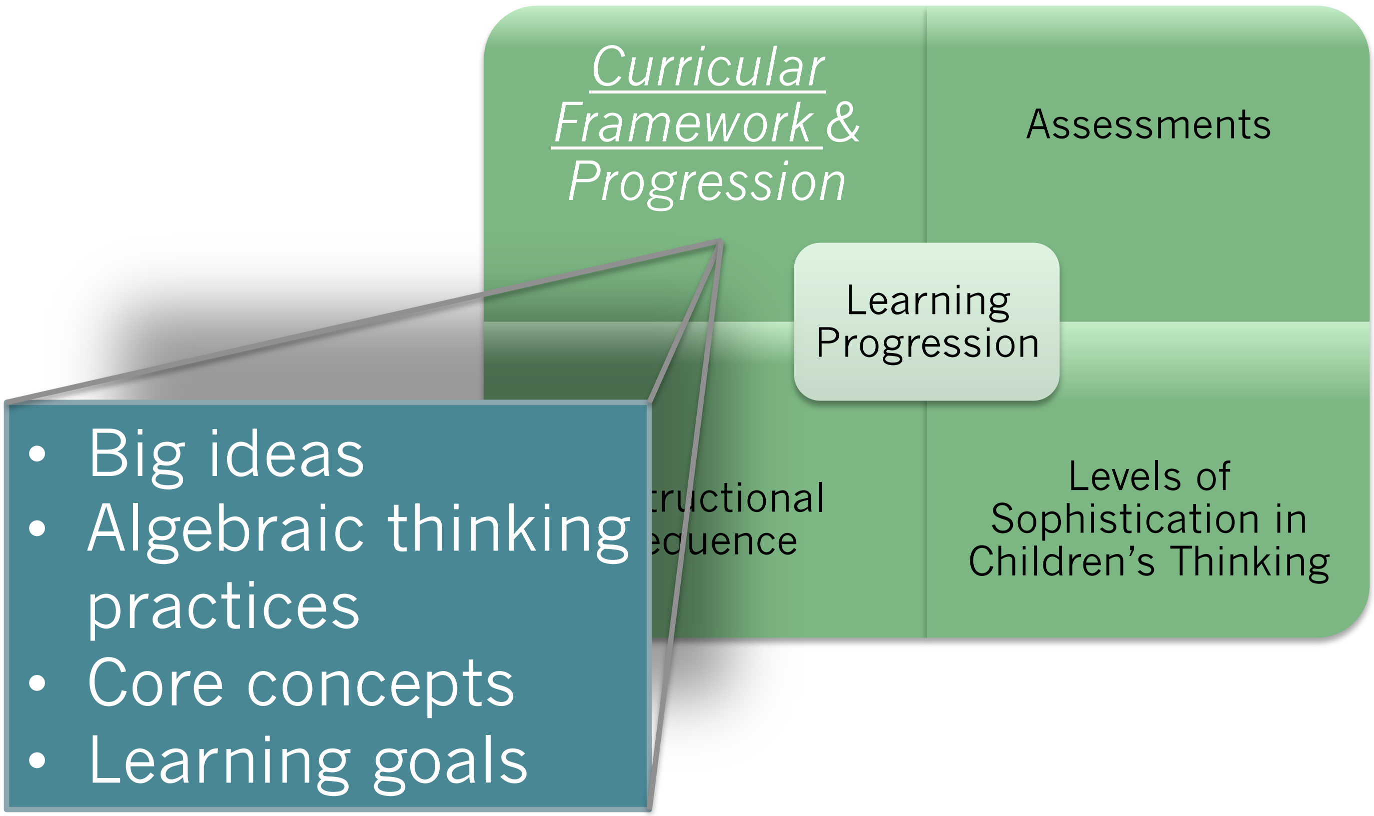
Learning

Instructional Sequence

Levels of Sophistication in Children's Thinking

Barrett & Battista (2014); Battista (2004); Clements & Sarama (2004);
Duncan & Hmelo-Silver (2009); Shin, Stevens, Short, Krajcik (2009)

An Early Algebra Learning Progression



Baroody, Cibulskis, Lai, & Li (2004); Clements & Sarama (2004); Shin, Stevens, Short, & Krajcik (2009)

Big Ideas

- Equations, expressions, equivalence, inequality
- Generalized arithmetic
- Functional thinking
- Variable
- Proportional reasoning

Algebraic Thinking Practices

- Generalizing relationships
- Representing generalizations
- Justifying generalizations
- Reasoning with generalizations

Blanton, Stephens, Knuth, Gardiner, Isler & Kim (2015); Kaput (2008)

Equivalence, Expressions, Equations, & Inequalities

"developing a relational understanding of the equal sign, representing and reasoning with expressions and equations in their symbolic form, and describing relationships between and among generalized quantities that may or may not be equivalent"

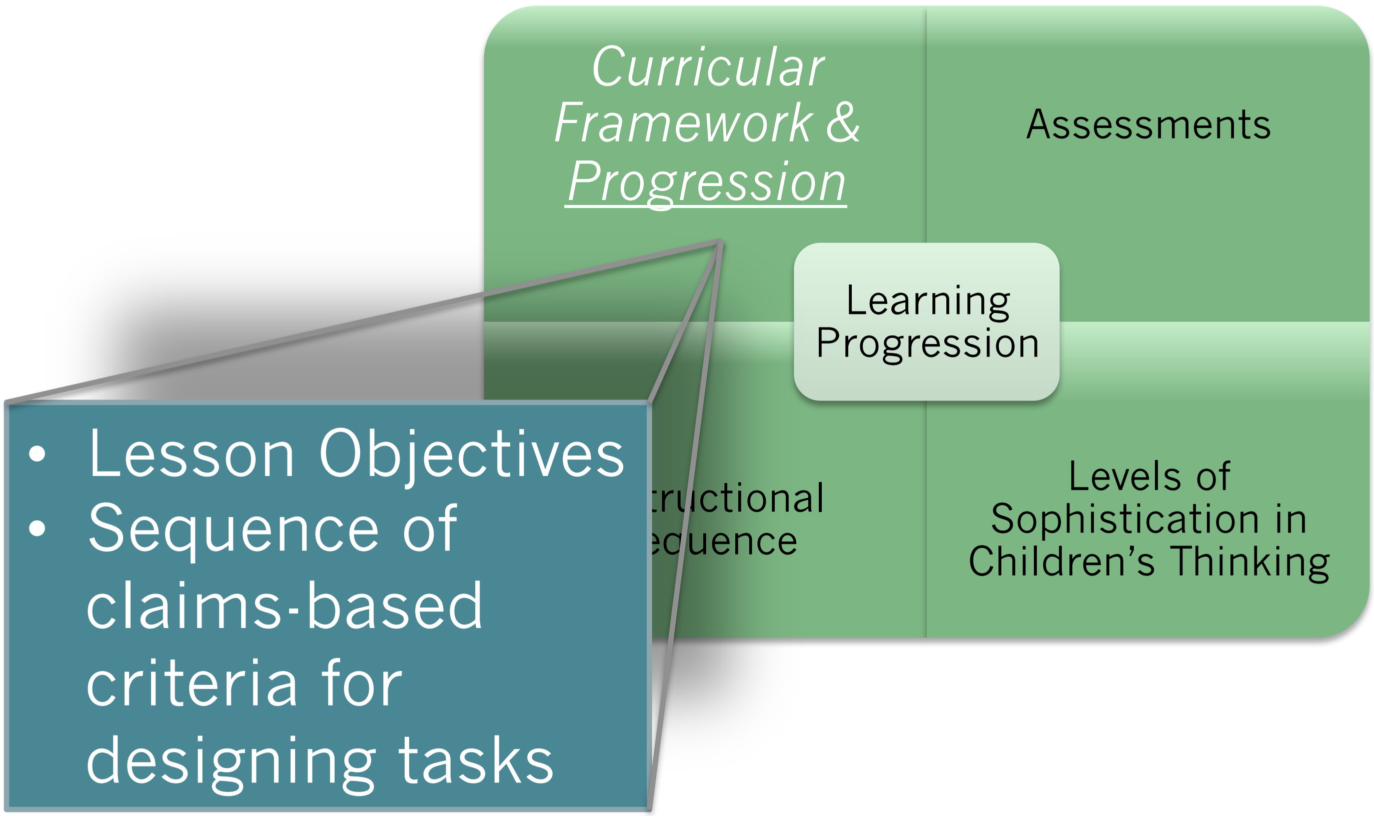
Blanton et al. (2015)

Equivalence, Expressions, Equations, & Inequalities

Core Concept	Learning Goals
2.1) The equal sign is used to represent the equivalence of two quantities or mathematical expressions.	• Understand the equal sign as a relational (rather than operational) symbol. • Interpret meaning of equations represented in symbolic form (e.g., understand that a = a represents that the measure of a quantity is equal to itself)

Blanton et al. (2015); Battista (2004); Clements & Sarama (2004); Shin, Stevens, Short, & Krajcik (2009)

An Early Algebra Learning Progression



Clements & Sarama (2004); Shin, Stevens, Short, & Krajcik (2009)

Lesson Objectives

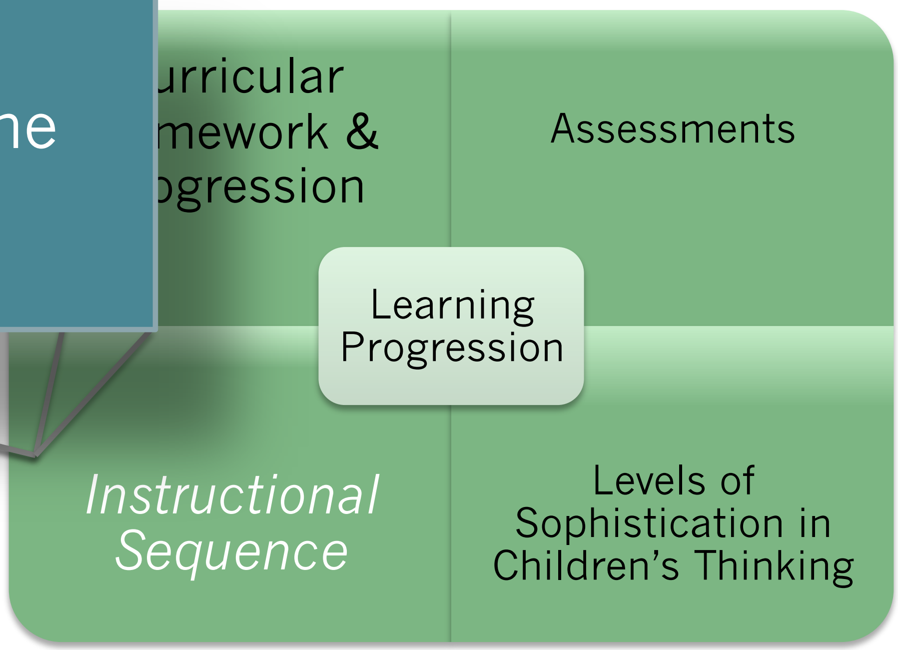
Develop a relational understanding of the equal sign by solving equations for missing values. Solutions may be obtained by identifying and reasoning with structural relationships in the equation or by using arithmetic strategies. (Grade 3 Lesson 2)

Understand how to express the relationship between two equivalent expressions using an equation. (Grade 3 Lessons 9-10)

	Grade 3 Lessons																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
bjective	•	•					•	•	•	•								
mp Start	•	•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	

An Early Algebra Learning Progression

- Tasks for Students
- Support for the Practice of Teaching



Clements & Sarama (2004); Shin, Stevens, Short, & Krajcik (2009)

Jumpstart

Are the following equations true or false? Explain.

12 + 8 = 20 + 5

34 = 20 + 14

5 = 5

Understanding the equal sign

A. What numbers will make the following equations true?

4 + 6 = ___ + 6

4 + 7 = ___ + 8

28 + 3 = ___ + 2

28 + 15 = ___ + 14

9 + ___ = 8 + 4

8 = ___

0 + ___ = 21

B. Use words, numbers, or pictures to show how you would find the missing value in the equation

15 + 20 = 14 + _____.

Review and Discuss

Find the missing value in the following equation. Explain.

100 + ___ = 101 + 52

Targeted Student Thinking

"The equal sign means that whatever amount you have on the left you must have on the right, therefore I know the missing value is…."

"In the equation 28 + 15 = ___ + 14, I know the missing value is 29 because 14 is one less than 15 so I knew I had to add 1 to the 28".

"12 + 8 = 20 + 5 is a false equation because the total on the left side of the equation is 20 and the total amount on the right side is 25, so the equation is not balanced."

Common Difficulties

Students who maintain an operational view of the equal sign after Lesson 1 may reason that the missing value in 28 + 15 = ___ + 14 is 43 because they simply operate on the equation from left to right, ignoring the '+ 14' on the right side of the equation.

TERC

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

RATIONALE FOR THE TASKS included in this lesson:

Focus on relationships

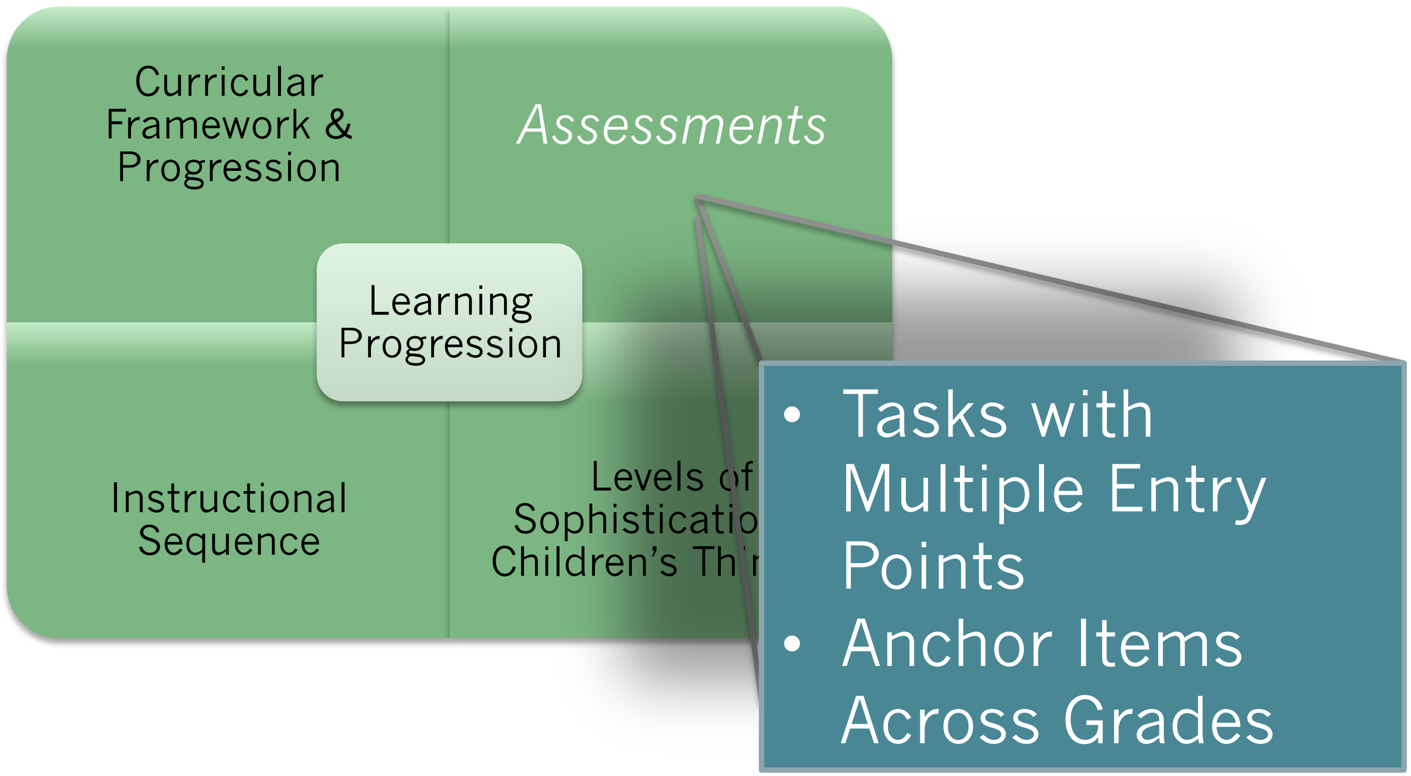
- The tasks included in this lesson are designed to encourage students to look for relationships across the equal sign and eventually use compensation strategies when these strategies are more efficient.

  The first equation, 4 + 6 = ___ + 6, can reveal how students are thinking about the equal sign and about equations in general. If a student responds:

  - "10, because 4 + 6 = 10," this indicates the student still holds an operational view of the equal sign.
  - "4, because on the left side 4 + 6 = 10, so I had to figure out what plus 6 would make 10 on the right side," this indicates the student views the equal sign relationally but is treating the two sides of the equation separately.
  - "4, because then the two sides of the equation are both 4 + 6," this indicates the student both views the equal sign relationally and is able to view the equation holistically, look for relationships across the equal sign, and at least in some cases find missing values without computing.

Variation in Students' Conceptions of Equality

An Early Algebra Learning Progression



Battista (2004)

1. Fill in the blank with the value that makes the number sentence true.

   7 + 3 = _____ + 4

   Explain how you got your answer.

2. Circle True or False.

   a) 12 + 3 = 10 + 5 True False

   b) 57 + 22 = 58 + 21 True False

   c) 39 + 121 = 121 + 39 True False

   Explain how you got your answer.

8. Ian says that because 37 + 10 = 47, he knows that 37 + 10 – 24 = 47 – 24.

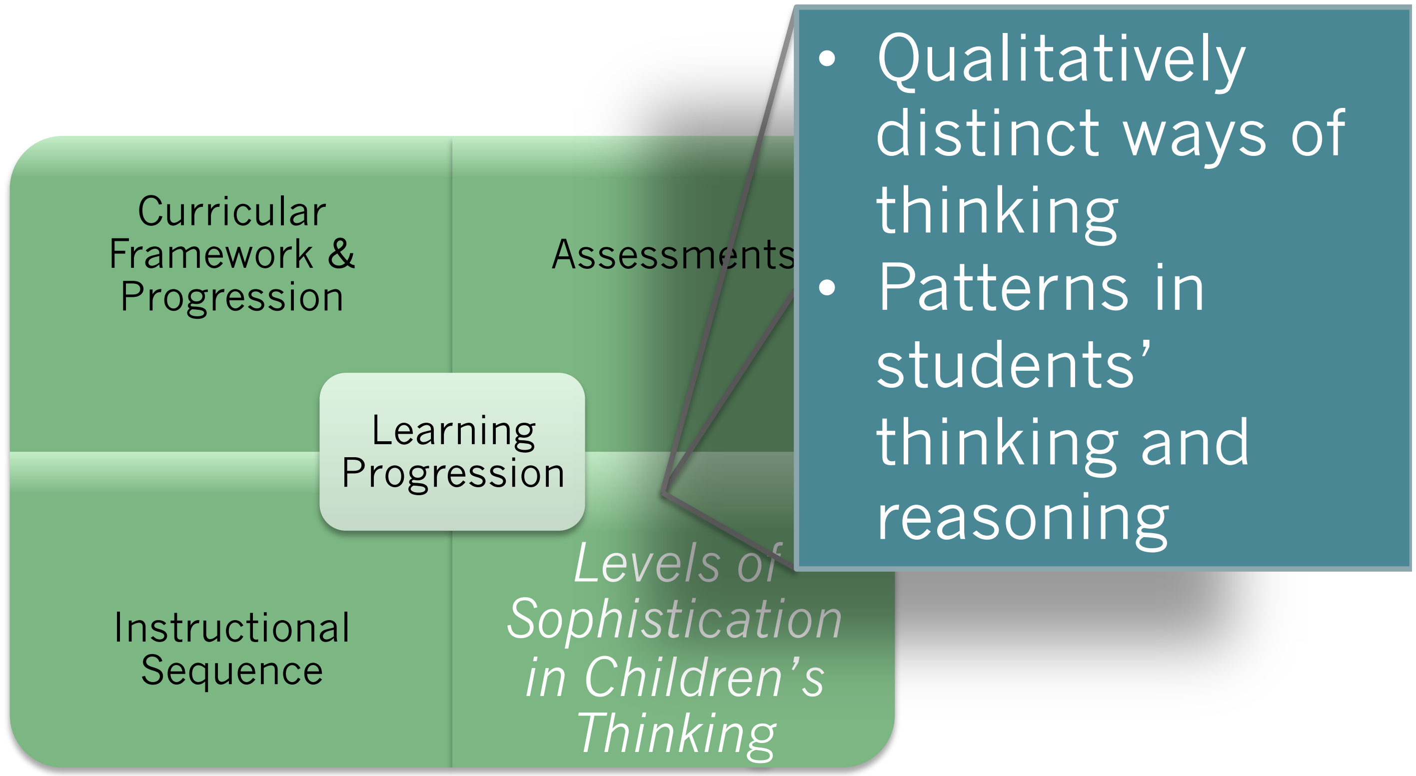
   a) Do you agree with Ian? Why or why not?

   b) Complete the following sentence using variables to represent Ian's thinking:

   If $a + b = c$, then ____.

Grade
3
4
5
6
7

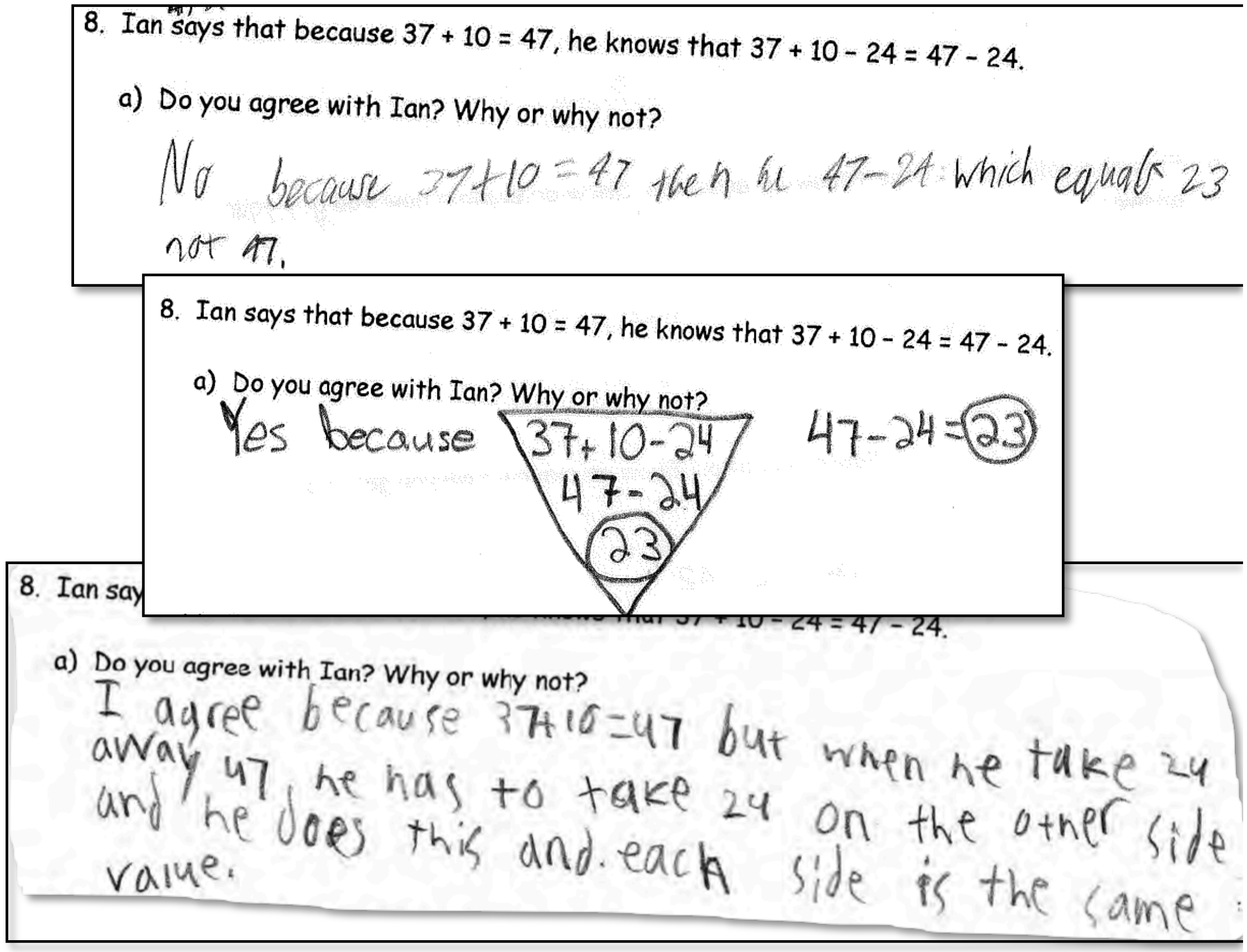
An Early Algebra Learning Progression



Battista (2004); Clements & Sarama (2014); Cobb et al. (2003)

Levels of Sophistication in Children's Thinking

Understanding Equality
Operational Views "=" as indicating "the answer"
Relational-Computational Interprets "=" as an equivalence relation between two computations.
Relational-Structural Interprets "=" as an equivalence relation between two expressions.



Rittle-Johnson, Matthews, Taylor, & McEldoon (2011)

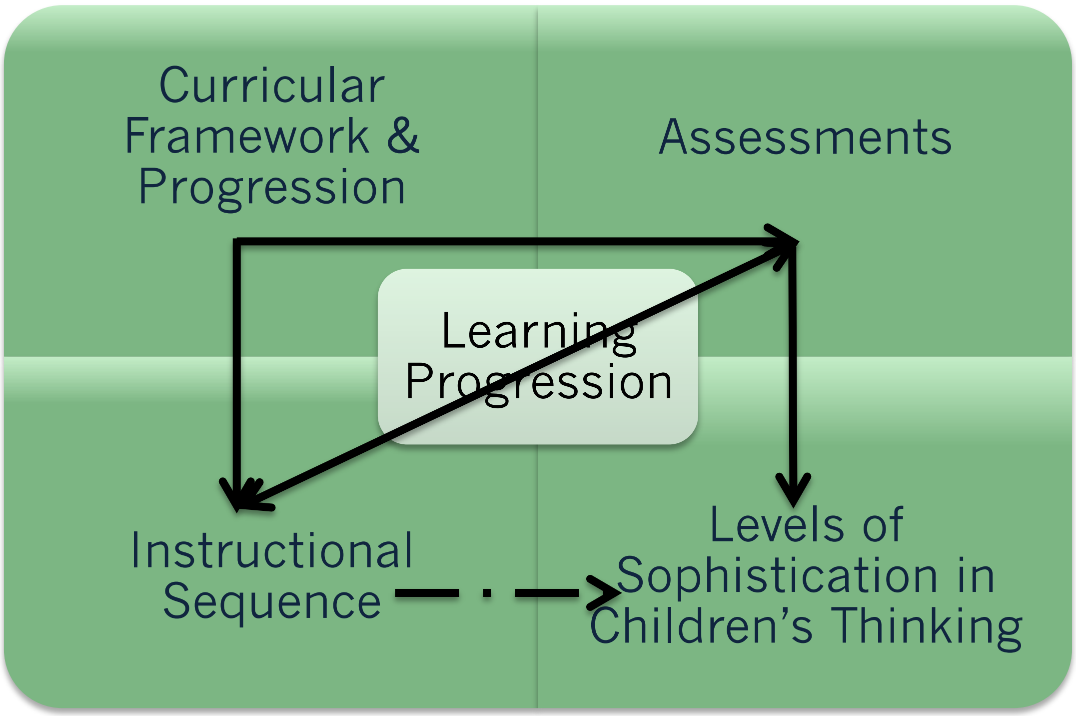
An Early Algebra Learning Progression

Curricular Framework & Progression	**Assessments**
• Big ideas & algebraic thinking practices • Core concepts • Learning goals • Sequence of lesson objectives	• Tasks with multiple entry points • Anchor items to measure across grades
Instructional Sequence • Tasks for students • Support for the practice of teaching	**Levels of Sophistication in Children's Thinking** • Qualitatively distinct ways of thinking • Patterns in students' thinking and reasoning

Battista (2004); Clements & Sarama (2004)

Discussion

How to prepare students in the elementary grades for success in middle grades algebra and beyond?



- Tool—research and development
- Course grain size—*years*
- Connections
- Methods and theoretical assumptions

Barrett & Battista (2014); Duncan & Hmelo-Silver (2009)

http://algebra.wceruw.org/

Nicole Fonger	Ana Stephens	Maria Blanton	Eric Knuth
Univ. of Wisconsin-Madison	Univ. of Wisconsin-Madison	TERC	Univ. of Wisconsin-Madison
nfonger @wisc.edu	acstephens @wisc.edu	maria_blanton @terc.edu	knuth @education. wisc.edu

Abstract

We detail a learning progressions approach to early algebra research and how existing work around learning progressions and trajectories in mathematics and science education has informed our development of a theoretical framework consisting of: a curricular progression of learning goals across big ideas; an instructional sequence of tasks based on objectives concerning content and algebraic thinking practices; assessments and coding schemes; and levels of sophistication in children's reasoning about algebraic concepts within big ideas of early algebra. We pay attention to the way we use terminology associated with learning progressions and trajectories and connections among student reasoning, curriculum, and instruction.

Big Idea Equivalence, Expressions, Equations, Inequalities

Algebraic Thinking Practices Representing and Reasoning with Genearlizations

Core Concept "The equal sign is used to represent the equivalence of two quantities or mathematical expressions."

Learning Goal "Understand the equal sign as a relational (rather than operational) symbol"

Lesson Objective "Develop a relational understanding of the equal sign by solving equations for missing values. Solutions may be obtained by identifying and reasoning with structural relationships in the equation or by using

Instructional Sequence
"Are the equations true or false?"
"Which numbers make the sentence true?"
Targeted Student Thinking
Common Student Difficulties

Assessments
"Open number sentences"
"True/false number sentences"
"Ian task—is it true? How do you know?"

Levels of Sophistication in Children's Thinking
"Operational"
"Relational-Computational"
"Relational-Structural"

Learning Over Time

Early Algebra Learning Progression
Curricular Framework and Progression
Instructional Sequence
Assessment Items
Levels of Sophistication in Children's Thinking

Our approach to *Learning Progressions* integrates curriculum, instruction, assessment, and sophistication in levels of students' understanding.

A theoretical frame and tool for designing and testing research-based *supports* for student learning and resulting *sophistication in students' reasoning*

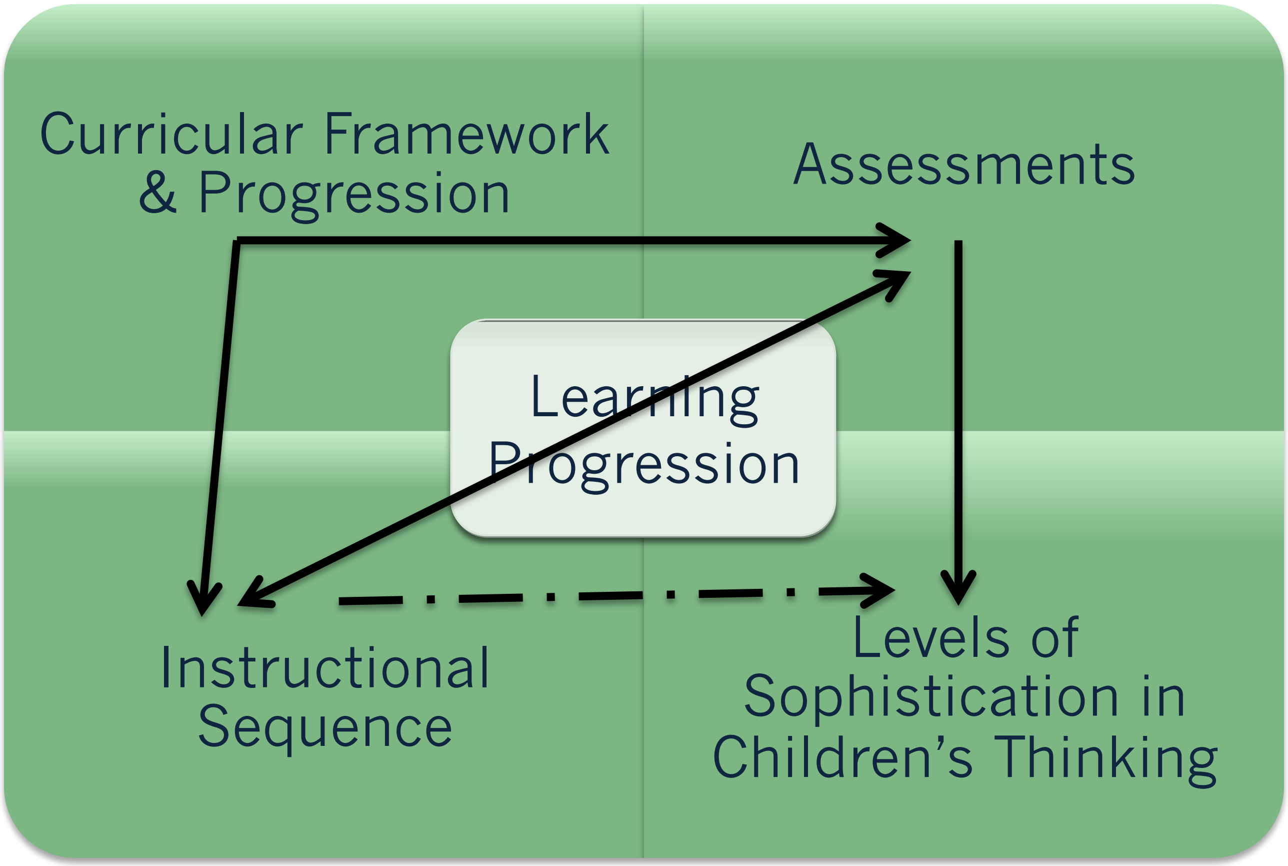
Contributions

- A theoretical frame and tool for designing *supports* for student learning and investigating resulting *sophistication in students' thinking*
- The EALP represents a course grain size across big ideas; students' developing ways of thinking over *years*

Future Directions

- Explicate theoretical assumptions and methodological approach to elucidate *connections*
- Investigate tighter links among supports and learning at smaller grain sizes

Connections



Barrett & Battista (2014); Duncan & Hmelo-Silver (2009)